A Configuration-Oriented SPICE Model for Multiconductor Transmission Lines in an Inhomogeneous Medium

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***Abstract*—A configuration-oriented SPICE model for multiple coupled lines in an inhomogeneous medium is presented in this paper. The circuit model consists of a network of uncoupled transmission lines and is readily modeled with simulation tools like LIBRA and SPICE, and provides an equivalent-circuit representation which is simple and topologically meaningful as compared to the model based on modal decomposition. This configuration-oriented model is derived by decomposing the immittance matrices associated with an n coupled-line $2n$-port system. Time- and frequency-domain simulations of typical coupled-line multiports are included to exemplify the utility of the model. The model is useful for the simulation and design of general single and multilayer coupled-line components such as filters and couplers and investigation of signal integrity issues, including crosstalk in interconnects associated with high-speed digital- and mixed-signal electronic modules and packages.**

***Index Terms*— Circuit modeling, coupled transmission lines, crosstalk, frequency-domain analysis, microstrip filters, SPICE, time-domain analysis, transmission lines.**

I. Introduction

THE analysis and modeling of coupled transmission systems including multiconductor transmission lines has been a topic of considerable interest in recent years. Advances in planar and layered interconnect and propagation structures and components in microwave, and high-speed digital- and mixed-signal circuits has resulted in increased interest in efficient accurate analysis and design of these circuits and systems. The circuit simulation and design of these structures is normally based on the characteristic parameters derived from a rigorous frequency-dependent electromagnetic (EM) solution or the line constants derived from quasi-static solutions. The quasi-static solutions lead to the $[R]$, $[L]$, $[G]$, and $[C]$ matrices associated with the multiconductor system. The frequency-dependent full-wave solutions lead to the computation of eigenvalues, eigenvectors, and eigenfunctions from which equivalent frequency-dependent elements of the $[R]$, $[L]$, $[G]$, and $[C]$ matrices can also be calculated [1], [2].

Several circuit models based on the solution of coupled transmission-line equations have been proposed in the past. For the case of lossless lines with frequency-independent line constants, a SPICE model based on modal decompositions was proposed in [3]. This model represents the congruent transformer bank [4] by dependent sources and leads to a circuit model consisting of linear-dependent sources and ideal delay elements representing uncoupled transmission lines. Simplified versions of the model valid for special cases of homogeneous electrically identical lines with near-neighbor coupling only have also been reported [5], [6]. A rigorous procedure leading to the configuration-oriented equivalent-circuit model, consisting of a system of transmission lines only, was reported for the case of homogeneous media in [7]. Similar useful models valid for special cases of inhomogeneous structures have also been proposed and used in the design of coupled-line circuits [8]–[11].

In this paper, the configuration-oriented SPICE model for the general case of inhomogeneous multilayer multiconductor lines is reported. The model consists of a system of transmission lines and has simpler SPICE input data requirements as compared to the modal decomposition-based models. It is shown that this configuration-oriented model can, in general, be implemented for the simulation of lossy and dispersive multiconductor inhomogeneous structures. The derivation of the circuit model is based on decomposition of the admittance or impedance matrix of the n coupled-line $2n$-port system. Closed-form expressions for the model parameters for the important cases of asymmetric coupled lines and symmetrical three coupled-line structures are included in this paper. Time- and frequency-domain simulation results for typical structures are presented to demonstrate the applications of the configuration-oriented SPICE models.

II. Equivalent-Circuit Model

The configuration-oriented SPICE model consists of a network of uncoupled transmission lines characterized by their propagation constants and impedances. The model can be readily derived from the admittance (impedance) matrix characterizing the $2n$-port system, as described in this section.

The procedure for deriving the expression for the admittance or impedance matrix of the general $2n$-port is well known and is based on the solution of coupled transmission-line equations

$$\frac{\partial v]}{\partial z} = -[Z(\omega)]i] \tag{1}$$

$$\frac{\partial i]}{\partial z} = -[Y(\omega)]v] \tag{2}$$

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where vectors $v]$ and $i]$ represent voltages and currents on the lines and

$$[Z(\omega)] = [R] + j\omega[L] \quad [Y(\omega)] = [G] + j\omega[C]$$

$[R]$, $[L]$, $[G]$, $[C]$ are the per-unit-length line constant matrices whose elements are, in general, frequency dependent. The coupled transmission-line equations (1) and (2) are decoupled with the help of voltage and corresponding current eigenvector matrices $[M_V]$ and $[M_I]$ ($[M_I] = [M_V]^{-T}$), respectively, leading to the characterization of the general n lines $2n$-port by its admittance matrix [12] as given by (see part A in the Appendix)

$$[Y] = \begin{bmatrix} [Y_A] & [Y_B] \\ [Y_B] & [Y_A] \end{bmatrix} \tag{3}$$

with

$$\begin{aligned} [Y_A] &= [Y_{\rm LM}] * [M_V][\coth(\gamma_i l)]_{\rm diag}[M_I]^T \\ [Y_B] &= [Y_{\rm LM}] * [M_V][-{\rm csch}(\gamma_i l)]_{\rm diag}[M_I]^T \\ [Y_{\rm LM}]_{n\times n} &= \begin{bmatrix} Y_{{\rm LM}_{11}} & Y_{{\rm LM}_{12}} & \cdot & Y_{{\rm LM}_{1n}} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ Y_{{\rm LM}_{n1}} & \cdot & \cdot & Y_{{\rm LM}_{nn}} \end{bmatrix} \end{aligned}$$

where γ_i represents the ith eigenvalue and is the ith normal-mode propagation constant. $[Y_{\rm LM}]$ is the line-mode admittance matrix whose element $Y_{{\rm LM}_{km}}$ represents the characteristic admittance of the kth line for mth mode and l is the length of the uniform coupled multiconductor system. The operator "$*$" was defined in [12] for $[C] = [A] * [B]$, as a product of corresponding terms of matrices $[A]$ and $[B]$. It is readily shown that the admittance matrix of the $2n$-port, as given by (3), can be decomposed as

$$[Y] = \sum_{m=1}^{n} [Y_m]. \tag{4}$$

The $[Y_m]$ represents the partial admittance matrix of the $2n$-port corresponding to mode m and can be expressed as

$$[Y_m]_{2n\times 2n} = \begin{bmatrix} \coth(\gamma_m l)[Y_{\rm ch}^m] & -{\rm csch}(\gamma_m l)[Y_{\rm ch}^m] \\ -{\rm csch}(\gamma_m l)[Y_{\rm ch}^m] & \coth(\gamma_m l)[Y_{\rm ch}^m] \end{bmatrix} \tag{5}$$

where

$$[Y_{\rm ch}^m]_{n\times n} = ([Y_{\rm LM}] * [M_V])[D_m]_{\rm diag}[M_I]^T \tag{6}$$

and

$$[D_m]_{\rm diag} = \begin{Bmatrix} D_m(j,j) = 0, & j \neq m \\ D_m(j,j) = 1, & j = m \end{Bmatrix}. \tag{7}$$

The symmetric matrix $[Y_{\rm ch}^m]$ (see part B in the Appendix) defined in (5) and (6) corresponds to the characteristic admittance matrix for mode m, and its (i,j)th element is given by

$$Y_{\rm ch}^m(i,j) = Y_{\rm LM}(i,m)M_V(i,m)M_I^T(m,j). \tag{8}$$

The matrix $[Y_m]$ in (5) is similar to the $2n$-port admittance matrix corresponding to n coupled lines in a homogeneous medium. Recalling that the admittance matrix of a transmission line two-port having length l, propagation constant γ, and characteristic admittance Y_c is given as

$$[Y]_{2\times 2} = \begin{bmatrix} Y_c \coth(\gamma l) & -Y_c\,{\rm csch}(\gamma l) \\ -Y_c\,{\rm csch}(\gamma l) & Y_c \coth(\gamma l) \end{bmatrix}. \tag{9}$$

Symmetry of $[Y_{\rm ch}^m]$ implies that the $2n$-port network represented by $[Y_m]$ consists of these transmission lines connected by a so-called "configuration-oriented" manner [7]. That is, the partial admittance matrix $[Y_m]$ for mode m is synthesized by a homogeneous configuration-oriented model [7] having transmission-line electrical lengths corresponding to the mth-mode eigenvalue. The complete network is then obtained as a parallel combination of the n, $2n$ ports with each $2n$-port corresponding to an orthogonal mode. A similar procedure can be applied to the impedance matrix leading to a dual topology and corresponding network of transmission lines that is equivalent to the multiconductor multiport. It is seen that the admittance or impedance matrix of an n multiconductor transmission-line system can, in general, be simulated by $n^2(n+1)/2$ transmission lines. In the case of symmetry, the number of lines are reduced depending upon the type of symmetry.

An alternate approach of deriving these circuits involves the use of the characteristic impedance or admittance matrix of the coupled system. These matrices represent a network which simultaneously terminates all the modes on all the lines. The elements of these matrices represent the characteristic immittance of the transmission lines that constitute the equivalent circuit. The length of the transmission lines corresponds to the electrical length of the corresponding mode. Both the $2n$-port and characteristic immittance-matrix-based decomposition procedures, of course, lead to the same equivalent-circuit representation (see part C in the Appendix).

III. Network Model for Uniform Coupled Lines

The procedure presented in Section II is applied to general asymmetric and symmetric three-line cases to illustrate the technique and derive closed-from expressions for the modal parameters for these two important cases.

A. Asymmetric Coupled Lines

A general uniform coupled two-line system can be decoupled in terms of two propagating modes π and c [13]. The voltage and corresponding current eigenvector matrices $[M_V]$, $[M_I]$ can be defined as ([13])

$$[M_V] = \begin{bmatrix} 1 & 1 \\ R_c & R_\pi \end{bmatrix}$$

and

$$[M_I] = ([M_V]^{-1})^T = \frac{1}{R_\pi - R_c}\begin{bmatrix} R_\pi & -R_c \\ -1 & 1 \end{bmatrix} \tag{10}$$

where γ_π and γ_c represent eigenvalues and are the normal-mode propagation constants for the π and c modes, and R_π and R_c represent the ratio of voltage on line 2 with respect to line 1 for π and c modes, respectively [13]. The line-mode admittance matrix $[Y_{\rm LM}]$ is expressed as

$$[Y_{\rm LM}]_{2\times 2} = \begin{bmatrix} Y_{c1} & Y_{\pi 1} \\ Y_{c2} & Y_{\pi 2} \end{bmatrix}. \quad (11)$$

The admittance matrix of the general asymmetric two-line system [13] obtained from (3), (10), and (11) is expressed as a sum of two matrices corresponding to the two modes using (4)–(8), and are found to be

$$[Y]_{4\times 4} = \begin{bmatrix} \coth(\gamma_\pi l)[Y^\pi_{\rm ch}] & -{\rm csch}(\gamma_\pi l)[Y^\pi_{\rm ch}] \\ -{\rm csch}(\gamma_\pi l)[Y^\pi_{\rm ch}] & \coth(\gamma_\pi l)[Y^\pi_{\rm ch}] \end{bmatrix}_\pi + \begin{bmatrix} \coth(\gamma_c l)[Y^c_{\rm ch}] & -{\rm csch}(\gamma_c l)[Y^c_{\rm ch}] \\ -{\rm csch}(\gamma_c l)[Y^c_{\rm ch}] & \coth(\gamma_c l)[Y^c_{\rm ch}] \end{bmatrix}_c \quad (12)$$

where

$$[Y^\pi_{\rm ch}] = \begin{bmatrix} \dfrac{R_c Y_{\pi 1}}{R_c - R_\pi} & \dfrac{-Y_{\pi 1}}{R_c - R_\pi} \\ \dfrac{Y_{\pi 2}}{(R_c - R_\pi)/R_c R_\pi} & \dfrac{-R_\pi Y_{\pi 2}}{R_c - R_\pi} \end{bmatrix}$$

and

$$[Y^c_{\rm ch}] = \begin{bmatrix} \dfrac{-R_\pi Y_{c1}}{R_c - R_\pi} & \dfrac{Y_{c1}}{R_c - R_\pi} \\ \dfrac{-Y_{c2}}{(R_c - R_\pi)/R_c R_\pi} & \dfrac{R_c Y_{c2}}{R_c - R_\pi} \end{bmatrix}. \quad (13)$$

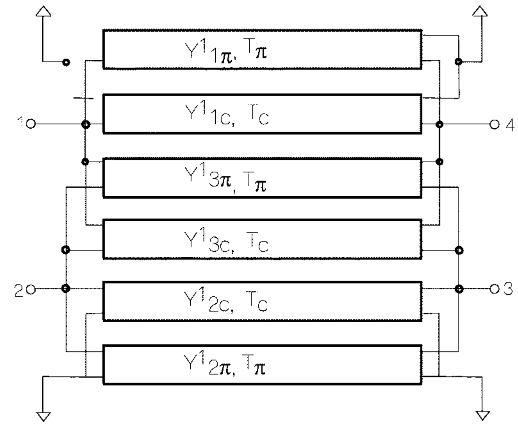
Each matrix $[Y]_\pi$ and $[Y]_c$ in (12) can be synthesized, in general, by three transmission lines connected at the input and output ends in a π configuration [7]. These two networks connected in parallel yield a complete equivalent circuit, as shown in Fig. 1(a), for the coupled-line system. The expressions for the characteristic admittance of the transmission lines in the SPICE model in Fig. 1(a) correspond to the admittance matrices given by (13) and are given by [see part A in the Appendix, (37) and (38)],

$$\begin{aligned} Y^1_{1\pi} &= \frac{Y_{\pi 1}(R_c - 1)}{R_c - R_\pi} \\ Y^1_{2\pi} &= \frac{R_\pi Y_{\pi 2}(R_c - 1)}{R_c - R_\pi} \\ Y^1_{3\pi} &= \frac{Y_{\pi 1}}{R_c - R_\pi} \\ Y^1_{1c} &= \frac{Y_{c1}(1 - R_\pi)}{R_c - R_\pi} \\ Y^1_{2c} &= \frac{R_c Y_{c2}(1 - R_\pi)}{R_c - R_\pi} \\ Y^1_{3c} &= \frac{-Y_{c1}}{R_c - R_\pi}. \end{aligned} \quad (14)$$

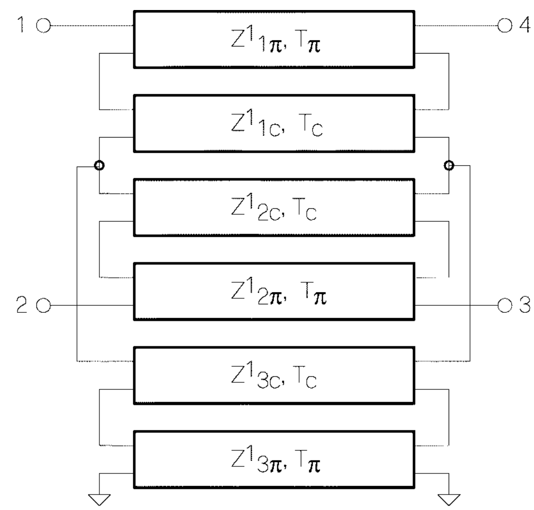
As noted earlier, the impedance matrix can also be used to construct an alternate equivalent model of the coupled system. The impedance matrix of a general asymmetric coupled-line system given in [13] is readily decomposed into two modes

$$[Z]_{4\times 4} = [Z_\pi]_{4\times 4} + [Z_c]_{4\times 4}. \quad (15)$$

Each matrix in (15) can be modeled by three transmission lines connected in a T configuration for each mode. The complete equivalent circuit is shown in Fig. 1(b). The expressions for



(a)



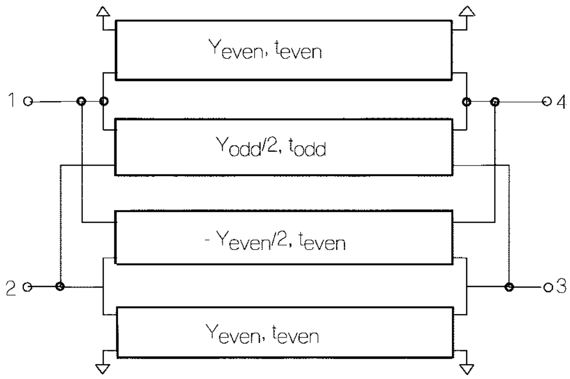
(b)

Fig. 1. (a) SPICE model for asymmetric coupled lines based on a four-port admittance matrix. (b) SPICE model based on four-port impedance matrix. (T_c and T_π are the time delays associated with the c and π modes. In case of lossy coupled lines, the delays and characteristic admittances of uncoupled transmission lines are function of frequency.)

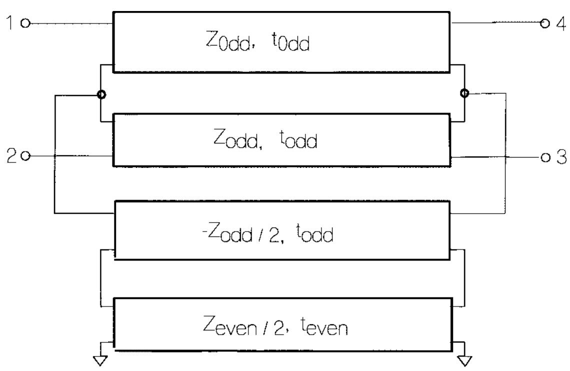
characteristic impedance of the transmission lines in this SPICE model [see Fig. 1(b)] are given by

$$\begin{aligned} Z^1_{1\pi} &= \frac{Z_{\pi 1} R_c}{R_c - R_\pi}(1 - R_\pi) \\ Z^1_{2\pi} &= \frac{Z_{\pi 2}}{R_c - R_\pi}(1 - R_\pi) \\ Z^1_{3\pi} &= \frac{Z_{\pi 1} R_c R_\pi}{R_c - R_\pi} \\ Z^1_{1c} &= \frac{Z_{c1} R_\pi}{R_c - R_\pi}(R_c - 1) \\ Z^1_{2c} &= \frac{Z_{c2}}{R_c - R_\pi}(R_c - 1) \\ Z^1_{3c} &= -\frac{Z_{c1} R_c R_\pi}{R_c - R_\pi}. \end{aligned} \quad (16)$$

It should be noted that the characteristic impedance and admittance matrices of the coupled-line system are symmetric and represent a passive n-port network. However, the characteristic impedance of some transmission lines in the equivalent circuit can be negative depending on modal decom-



(a)



(b)

Fig. 2. (a) SPICE model for symmetrical coupled lines based on a four-port admittance matrix. (b) SPICE model based on four-port impedance matrix. ($t_{\rm even}$ and $t_{\rm odd}$ are the time delays associated with the even and odd modes. In case of lossy coupled lines, the delays and characteristic admittances of uncoupled transmission lines are function of frequency.)

position. The decomposition of the characteristic admittance matrix (stable and passive network) into the sum of n partial admittance matrices leads to a stable configuration-oriented model in spite of the fact that some of the elements are not passive. This is like having two equal-length lines in parallel. For a stable system, one of them can have negative characteristic admittance as long as the total combined characteristic admittances of the two lines is positive. These negative-impedance transmission lines can be simulated by positive-impedance transmission-line elements with the linear dependent sources for the impedance conversion or directly as negative-impedance transmission lines on many computer-aided design (CAD) tools like LIBRA. Furthermore, in case of symmetric lossless coupled lines $R_\pi = -1$, $R_c = 1$, $Y_{\pi 1} = Y_{\pi 2}$, and $Y_{c1} = Y_{c2}$ leads to $Y_{\rm odd} = 2Y_{3\pi}^1$ and $Y_{\rm even} = Y_{1c}^1$, and the model reduces to the four transmission-line system presented in [9] [see Fig. 2(a)] or to the T equivalent network, as shown in Fig. 2(b) ($Z_{\rm even} = 2Z_{3c}^1$ and $Z_{\rm odd} = Z_{1\pi}^1$). Similarly, for the case of lossless symmetric coupled lines in a homogeneous medium ($\gamma_\pi = \gamma_c$), the equivalent system reduces to a three-transmission-line system, as in [7].

B. Symmetric Three Coupled Lines

For the case of symmetric coupled three-line structures, the voltage and associated current eigenvector matrices corresponding to the three propagating modes a, b, and c are given by [14]

$$[M_V] = \begin{bmatrix} 1 & 1 & 1 \\ 0 & R_{v1} & R_{v2} \\ -1 & 1 & 1 \end{bmatrix} \tag{17}$$

and

$$[M_I] = ([M_V]^{-1})^T = \frac{1}{2(R_{v1} - R_{v2})} \cdot \begin{bmatrix} R_{v1} - R_{v2} & -R_{v2} & R_{v1} \\ 0 & 2 & -2 \\ R_{v2} - R_{v1} & -R_{v2} & R_{v1} \end{bmatrix}. \tag{18}$$

The line-mode admittance matrix $[Y_{\rm LM}]$ is

$$[Y_{\rm LM}]_{3\times 3} = \begin{bmatrix} Y_{a1} & Y_{b1} & Y_{c1} \\ Y_{a2} & Y_{b2} & Y_{c2} \\ Y_{a3} & Y_{b3} & Y_{c3} \end{bmatrix} \tag{19}$$

and γ_a, γ_b, γ_c are the propagation constants associated with the normal modes a, b, and c [14]. For three symmetric coupled lines $Y_{a1} = Y_{a3}$, $Y_{b1} = Y_{b3}$, and $Y_{c1} = Y_{c3}$. Substituting (17)–(19) in (6) leads to the characteristic admittance matrices for the modes a, b, and c, and are given in (39).

The six-port transmission-line network is then readily constructed as three modal networks connected in parallel. The complete network then consists of 13 transmission lines. The expressions for the characteristic admittances for the lines are given as follows [see (37) and (38), part C in the Appendix]:

$$[Y_{\rm ch}^a] = \begin{bmatrix} \frac{Y_{a1}}{2} & 0 & \frac{-Y_{a1}}{2} \\ 0 & 0 & 0 \\ \frac{-Y_{a3}}{2} & 0 & \frac{Y_{a3}}{2} \end{bmatrix}$$

$$[Y_{\rm ch}^b] = \frac{1}{2(R_{v1} - R_{v2})} \cdot \begin{bmatrix} -R_{v2}Y_{b1} & 2Y_{b1} & -R_{v2}Y_{b1} \\ -R_{v1}R_{v2}Y_{b2} & 2R_{v1}Y_{b2} & -R_{v1}R_{v2}Y_{b2} \\ -R_{v2}Y_{b3} & 2Y_{b3} & -R_{v2}Y_{b3} \end{bmatrix}$$

and

$$[Y_{\rm ch}^c] = \frac{1}{2(R_{v1} - R_{v2})} \cdot \begin{bmatrix} R_{v1}Y_{c1} & -2Y_{c1} & R_{v1}Y_{c1} \\ R_{v1}R_{v2}Y_{c2} & -2R_{v2}Y_{c2} & R_{v1}R_{v2}Y_{c2} \\ R_{v1}Y_{c3} & -2Y_{c3} & R_{v1}Y_{c3} \end{bmatrix}. \tag{20}$$

Transmission lines between ports and terminals are as follows.

- Ports 1 and 4:
$$\frac{Y_{b1}(1 - R_{v2})}{R_{v1} - R_{v2}} \quad \frac{Y_{c1}(R_{v1} - 1)}{R_{v1} - R_{v2}}.$$
- Ports 2 and 5:
$$\frac{Y_{b2}R_{v1}(1 - R_{v2})}{R_{v1} - R_{v2}} \quad \frac{Y_{c2}R_{v2}(R_{v1} - 1)}{R_{v1} - R_{v2}}.$$

- Ports 3 and 6:

$$\frac{Y_{b3}(1-R_{v2})}{R_{v1}-R_{v2}} \quad \frac{Y_{c3}(R_{v1}-1)}{R_{v1}-R_{v2}}.$$

- Terminal pairs (1, 2) and (4, 5):

$$\frac{-Y_{b1}}{R_{v1}-R_{v2}} \quad \frac{Y_{c1}}{R_{v1}-R_{v2}}.$$

- Terminal pairs (2, 3) and (5, 6):

$$\frac{R_{v1}R_{v2}Y_{b2}}{2(R_{v1}-R_{v2})} \quad \frac{-R_{v1}R_{v2}Y_{c2}}{2(R_{v1}-R_{v2})}.$$

- Terminal pairs (1, 3) and (5, 6):

$$\frac{Y_{a1}}{2} \quad \frac{R_{v2}Y_{b1}}{2(R_{v1}-R_{v2})} \quad \frac{-R_{v1}Y_{c1}}{2(R_{v1}-R_{v2})}.$$

The electrical length of these transmission lines are given by the corresponding electrical length of the three modes.

C. Lossy Dispersive Multiconductor Coupled Lines

In general, the coupled-line systems have conductor and dielectric losses and, therefore, the uncoupled transmission lines in the configuration-oriented model have frequency-dependent complex propagation constants and the complex characteristic impedances. The multiport SPICE simulation for lossy dispersive multiconductor coupled lines can be accomplished by modeling the uncoupled lines obtained from the formulation presented in Section II in terms of their two-port frequency-dependent network functions such as $[Y]$- or $[S]$-parameters or modeling them in terms of equivalent circuits consisting of ideal lumped and delay elements, as shown in [15], for a single microstrip. The frequency-dependent complex characteristic parameters (propagation constants, line-mode impedances, eigenvector matrices) are computed using a rigorous frequency-dependent technique like the spectral-domain approach [12] or by using CAD-oriented quasi-TEM methods, e.g., [16]. The model transmission-line parameters can be obtained in terms of equivalent frequency-dependent self and mutual line constants per unit length, as shown in [1]. As an example, for the simple case of identical coupled lines, the even- and odd-mode propagation constants and impedances are readily expressed as

$$\gamma_{e,o} = \sqrt{[(R\pm R_m)+j\omega(L\pm L_m)][(G\mp G_m)+j\omega(C\mp C_m)]} \tag{21}$$

and

$$Z_{e,o} = \sqrt{\frac{(R\pm R_m)+j\omega(L\pm L_m)}{(G\mp G_m)+j\omega(C\mp C_m)}} \tag{22}$$

in terms of the frequency-dependent equivalent-line constants representing self and mutual inductances, capacitances, resistances, and conductances per unit length of the lines. For the case of low losses, (21) and (22) can be simplified as

$$Z_{e,o} \approx \sqrt{\frac{L\pm L_m}{C\mp C_m}} \tag{23}$$

$$\gamma_{e,o} \approx j\omega\sqrt{(L\pm L_m)(C\mp C_m)} + \frac{R\pm R_m}{2Z_{e,o}} + \frac{G\mp G_m}{2}Z_{e,o}. \tag{24}$$

These line constants (R, R_m, L, L_m, G, G_m, C, C_m) can be calculated by using quasi-static as well as full-wave analysis of the coupled system [1], [12], [16]. For the general asymmetric coupled lines and multiconductor coupled systems, the modeling of individual lossy dispersive lines [15] in terms of network functions, frequency-dependent line constants, or equivalent circuits is similar to the symmetrical coupled-line case.

D. The Procedure

The general procedure for building a configuration-oriented SPICE model of general n-line $2n$-ports is summarized as follows.

1) Evaluate propagation constants (γ's), eigenvector matrices ($[M_I]$ or $[M_V]$) and line mode admittance matrix ($[Y_{\rm LM}]$) elements using full-wave analysis [1] or compute $[R]$, $[L]$, $[G]$, and $[C]$ matrices using quasi-static analysis [12] and then evaluate propagation constants (γ's) and a corresponding voltage eigenvector matrix using an eigenvalue equation and $[Y_{\rm LM}]$ line mode admittance matrix whose (k,m)th element corresponds to the characteristic admittance of the kth line for mth mode [12]–[14].
2) Using (4)–(8), decompose the admittance matrix, as in (3), into a sum of partial admittance matrices $[Y_m]$, each corresponding to the mth mode.
3) Each partial admittance matrix corresponding to a propagation mode is synthesized as a $2n$-port network with the two wire transmission lines, as in [7] [see part C in the Appendix, (37) and (38)], with the transmission line lengths equal to the electrical length of the respective mode.
4) Connect the $2n$-port networks of each partial admittance matrix in parallel to obtain the configuration-oriented SPICE model of a general n-line $2n$-port.

IV. Results

In order to validate the accuracy of the models and demonstrate their usefulness and versatility, the frequency- and time-domain responses of typical coupled-line structures are presented. Unlike most of the earlier circuit models, the present model can incorporate conductor, dielectric losses, and the dispersion due to this losses for the system of n coupled lines. The multiconductor n-line system can be represented as a $2n$-port (multiport) element, independent of termination conditions at those ports, enabling us to simulate the $2n$-port with linear and nonlinear terminations in a complex circuit environment.

Fig. 3 shows the time-domain response of a symmetrical lossless microstrip line terminated with the impedances given by z_t ($z_t = \sqrt{z_e z_o}$) at all the four ports. Due to the medium inhomogeneity, both the even and odd modes are excited and arrive at the terminating ports at different times, generating

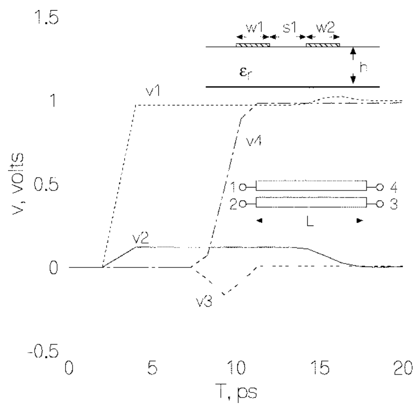


Fig. 3. Step response of the symmetric lossless coupled microstrip four-port. $\epsilon_r = 4.5$, $w1 = w2 = 0.18$ mm, $s1 = 0.04$ mm, $h = 0.1$ mm, $L = 1$ mm, strip thickness $= 12$ μm.

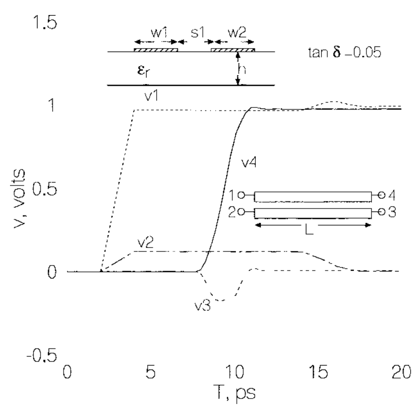


Fig. 4. Time-domain step response of the symmetric coupled microstrip four-port on FR-4 substrate. $\epsilon_r = 4.5$, $w1 = w2 = 0.18$ mm, $s1 = 0.04$ mm, $h = 0.1$ mm, $L = 1$ mm, strip thickness $= 12$ μm. Loss tangent of dielectric layer (tan δ) $= 0.05$.

far-end crosstalk. Intuitively, it is can be seen that in the equivalent circuit, for the even-mode excitation, the signal transmission is only through the transmission lines associated with the even mode. Similarly, for the odd-mode excitation, the signal transmission is through the lines associated with the odd mode. The response is exactly the same as that found by using the SPICE model based on the modal decomposition [3].

The time-domain step response of a lossy symmetrical coupled line having both dielectric and conductor losses is shown in Fig. 4. Due to the presence of these losses, the uncoupled transmission-line expressions given in (14) or (16) are frequency dependent. The approximate expression similar to (23) and (24) are used to simulate the individual (four) uncoupled lossy transmission lines given in (14) and (16). The simulations are performed using the configuration-oriented SPICE models ["π topology": Fig. 2(a) and "'T topology": Fig. 2(b)] with the HP-EEsof CAD tools. As expected, the time-domain waveforms obtained from both the π- and T-topology-based configuration-oriented SPICE models are found to be identical.

The time-domain step response of an asymmetric coupled lossless microstrip structure is shown in Fig. 5. As in the case above, the simulations are performed using the π-[(14): Fig. 1(a)] and the T-topology-based [(16): Fig. 1(b)] configuration-oriented SPICE models. The obtained time-

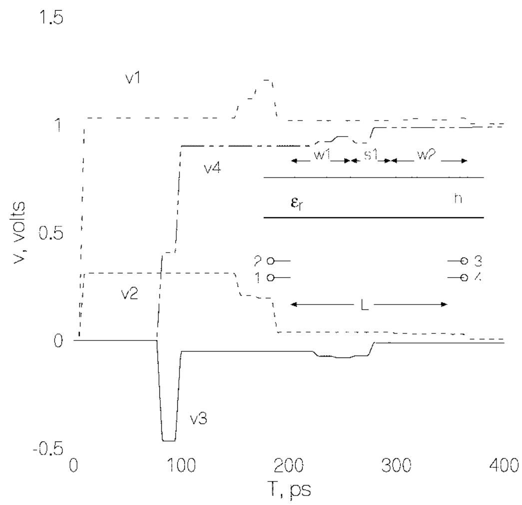


Fig. 5. Time-domain response of the asymmetric lossless coupled microstrip four-port. $\epsilon_r = 9.8$, $w1 = 0.24$ mm, $w2 = 0.48$ mm, $s1 = 0.04$ mm, $h = 1.0$ mm, $L = 9.3$ mm, strip thickness $= 12$ μm.

domain waveforms are found to be identical in both cases. Additionally, the asymmetric coupled microstrip four-port in this example is the same as that of [3] and validates the accuracy of the present model.

Fig. 6 shows the frequency response of edge-coupled two-section filter simulated by LIBRA using the coupled line model given in Fig. 1(a). Each section consists of asymmetric coupled microstrips. The effect of dielectric and conductor losses are also included in the simulation. The example demonstrates the application of the configuration-oriented model presented here for the design of general asymmetric and multiconductor coupled line circuits. The values of the $[R]$, $[G]$, $[L]$, and $[C]$ matrices for each asymmetric coupled line section in this filter are obtained by using a CAD-oriented quasi-TEM method [16]. The modeling of the individual lossy dispersive lines is done in terms of network functions [15]. The obtained response in Fig. 6 is similar to that of response obtained by the simulation of this filter using a full-domain EM tool (HP-EEsof, Momentum). The configuration-oriented model can, in general, include the frequency variations of $[R]$, $[G]$, $[L]$, and $[C]$ matrices and is dependent upon the availability of a single lossy dispersive transmission-line model, for which several good CAD-oriented techniques are already available.

The time-domain response for three symmetrical coupled lossless lines is shown in Fig. 7 in order to demonstrate the applicability of the model presented here for the simulation of multiconductor transmission lines in high-speed digital circuits. The input and output ports are terminated by 50-Ω impedances, except port 5, which drives a high-impedance CMOS inverter. The time-domain step response shows signal degradation, coupling, and crosstalk effects between the active and passive lines.

V. Conclusions

In conclusion, a new configuration-oriented SPICE model for multiple–coupled microstrip lines and other multiconductor structures in an inhomogeneous medium has been developed

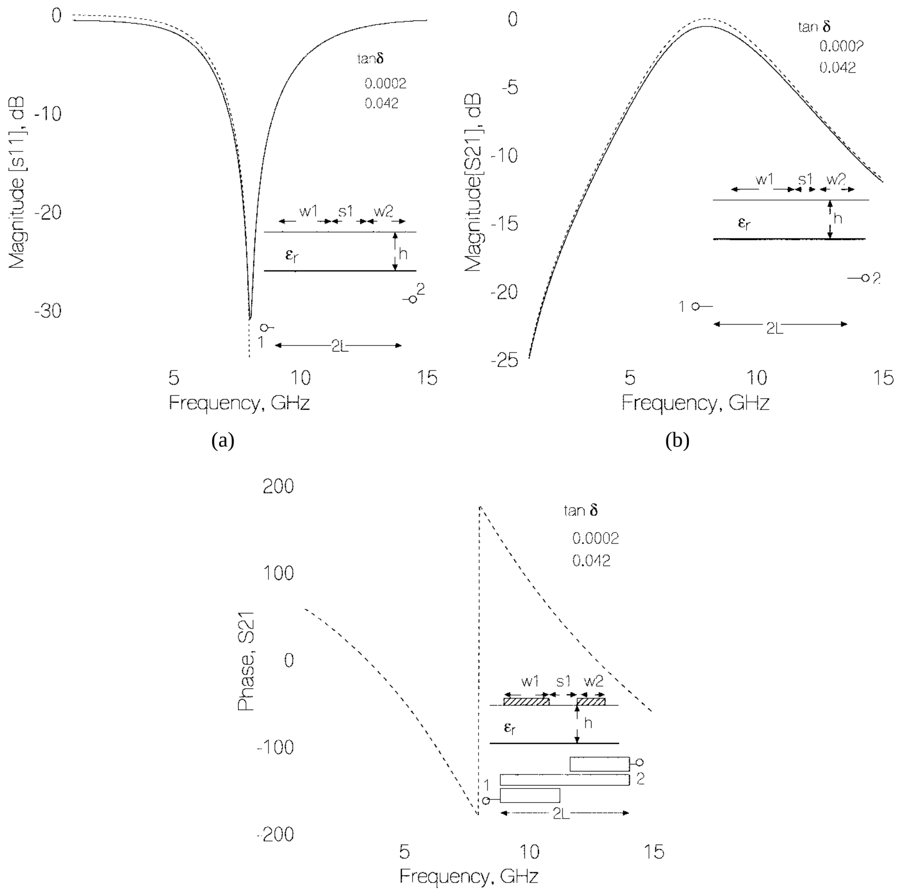


Fig. 6. Frequency response of a two-section asymmetric coupled microstrip filter on alumina for different dielectric losses (tan $\delta = 0.0002$ and $\tan\delta = 0.042$). (a) Magnitude of $S11$ in decibels. (b) Magnitude of $S21$ in decibels. (c) Phase response of $S21$. $\epsilon_r = 9.8$, $w1 = 0.4$ mm, $w2 = 0.25$ mm, $s1 = 0.04$ mm, $h = 0.63$ mm, $L = 3.75$ mm.

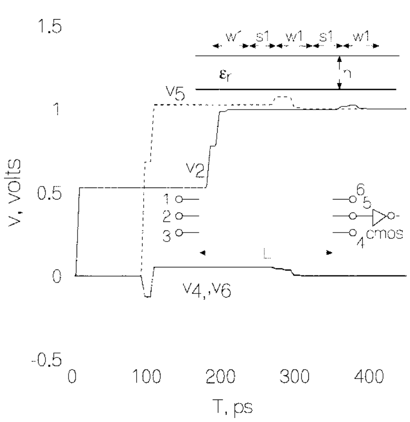


Fig. 7. Step response of the lossless three symmetric coupled microstrip six-ports (50-Ω termination at port 1, 2, 3, 4, 6 and port 5 terminated at high input impedance CMOS inverter. $\epsilon_r = 12.8$, $w1 = 0.3$ mm, $s1 = 0.3$ mm, $h = 0.635$ mm, strip thickness $= 12$ μm.

and presented. The model is based on the decomposition of the general n-line $2n$-port immittance matrix into a sum of partial-immittance matrices corresponding to each mode. The complete equivalent network is obtained by combining the resulting n $2n$-port configuration-oriented models synthesized for each partial immittance matrix. It is observed that n coupled multiconductor lines can, in general, be simulated $n^2(n+1)/2$ transmission lines. The configuration-oriented SPICE model provides a simple alternate equivalent network in terms of coupled multiconductor characteristic parameters, which are readily obtained from rigorous full-wave or quasi-TEM computations. This model is compatible with the simulation of lossy dispersive systems and should be quite helpful in the frequency- and time-domain simulation and design of multiconductor coupled systems. Several frequency- and time-domain simulation examples have been presented for typical interconnect and microwave component structures to demonstrate the applications of the model.

APPENDIX

A. Admittance Matrix: n Coupled-Line $2n$-Port System

A procedure similar to that in [13] leads to the solution of n coupled transmission-line equations [(1) and (2)] in terms of normal-mode parameters. Using this procedure, the expressions obtained for the voltages and corresponding currents in terms of $2n$ waves is given by

$$[v]_{n\times 1} = [M_V][e^{-\gamma_i x}]_{\text{diag}}[a_r]_{n\times 1} + [M_V][e^{\gamma_i x}]_{\text{diag}}[a_l]_{n\times 1} \quad (25)$$

$$[i]_{n\times 1} = ([Y_{\rm LM}] * [M_V])[e^{-\gamma_i x}]_{\rm diag}[a_r]_{n\times 1} - ([Y_{\rm LM}] * [M_V])[e^{\gamma_i x}]_{\rm diag}[a_l]_{n\times 1} \quad (26)$$

where the columns of matrix $[M_V]$ are the voltage eigenvectors, γ_i is the ith normal-mode propagation constant, $[Y_{\rm LM}]$ is the line-mode admittance matrix, and operator "$*$" is defined in (3). The admittance matrix for the multiconductor coupled-line $2n$-port system is obtained by combining (25) and (26). The final expression for the admittance matrix is given in (3).

B. Proof: $[Y_{\rm ch}^m]$ Is Symmetric

It is clear from (25) and (26) that in a system of n coupled lines, these $2n$ waves actually consist of two independent sets of n waves, each propagating in the opposite direction. Moreover, these n waves satisfy the coupled transmission-line equations (1) and (2), leading to

$$[Z][Y][M_V] = [M_V][\gamma_i^2]_{\rm diag} \quad (27)$$

and

$$[Y][Z]([Y_{\rm LM}] * [M_V]) = ([Y_{\rm LM}] * [M_V])[\gamma_i^2]_{\rm diag}. \quad (28)$$

Taking the transpose of (28) and observing that the matrix $[Z]$ and $[Y]$ are symmetric yields

$$([Y_{\rm LM}] * [M_V])^T[Z][Y] = [\gamma_i^2]_{\rm diag}([Y_{\rm LM}] * [M_V])^T. \quad (29)$$

The columns of matrix $[M_V]$ are the right eigenvectors and the rows of matrix $([Y_{\rm LM}] * [M_V])^T$ are left eigenvectors of matrix $[Z]$ $[Y]$. The equation obtained by multiplying on the right-hand side of (27) by $([Y_{\rm LM}]*[M_V])^T$ and then subtracting the equation obtained by multiplying on the left-hand side of (29) by $[M_V]$ is

$$([Y_{\rm LM}] * [M_V])^T[M_V][\gamma_i^2]_{\rm diag} - [\gamma_i^2]_{\rm diag} ([Y_{\rm LM}] * [M_V])^T[M_V] = 0. \quad (30)$$

Equation (30) shows that $([Y_{\rm LM}] * [M_V])^T[M_V]$ commutes with the diagonal matrix of distinct elements and, therefore, should be diagonal. Therefore,

$$([Y_{\rm LM}] * [M_V])^T[M_V] = [\lambda_i]_{\rm diag}. \quad (31)$$

The values of λ_i depend upon (1) and (2). Combining (31) with (6) gives

$$\begin{aligned}[Y_{\rm ch}^m] &= ([Y_{\rm LM}] * [M_V])[\lambda_i]_{\rm diag}^{-1}[D_m]_{\rm diag}([Y_{\rm LM}] * [M_V])^T \\ &= [M_V]^{-T}[\lambda_i]_{\rm diag}[D_m]_{\rm diag}[M_V]^{-1} \\ &= [Y_{\rm ch}^m]^T \end{aligned} \quad (32)$$

where the matrix $[D_m]_{\rm diag}$ is defined in (7).

C. The Procedure: Based on Characteristic Admittance or Impedance Matrix

An alternate approach can also be used in deriving the equivalent circuit with the help of the characteristic impedance matrix $[Z_{\rm ch}]$ or admittance matrix $[Y_{\rm ch}]$ ($[Y_{\rm ch}] = ([Y_{\rm LM}] * [M_V])[M_V]^{-1}$). For example, the characteristic admittance and impedance matrices for the coupled two-line structure, as given in [13], are

$$[Y_{\rm ch}] = \begin{bmatrix} \dfrac{-R_\pi Y_{c1} + R_c Y_{\pi 1}}{R_c - R_\pi} & \dfrac{Y_{c1} - Y_{\pi 1}}{R_c - R_\pi} \\ \dfrac{Y_{\pi 2} - Y_{c2}}{(R_c - R_\pi)/R_c R_\pi} & \dfrac{R_c Y_{c2} - R_\pi Y_{\pi 2}}{R_c - R_\pi} \end{bmatrix} \quad (33)$$

and

$$[Z_{\rm ch}] = \begin{bmatrix} \dfrac{Z_{\pi 1} R_c - Z_{c1} R_\pi}{R_c - R_\pi} & \dfrac{Z_{c1} - Z_{\pi 1}}{(R_\pi - R_c)/R_c R_\pi} \\ \dfrac{Z_{c2} - Z_{\pi 2}}{R_c - R_\pi} & \dfrac{R_c Z_{c2} - R_\pi Z_{\pi 2}}{R_c - R_\pi} \end{bmatrix} \quad (34)$$

where R_c and R_π are the ratio of voltage on conductor 2 to the voltage on conductor 1 for the two modes, Y_{c1}, Y_{c2}, $Y_{\pi 1}$, $Y_{\pi 2}$ are corresponding line mode admittances and Z_{c1}, Z_{c2}, $Z_{\pi 1}$, $Z_{\pi 2}$ are the line mode impedances.

The above impedance or admittance matrix for two coupled lines can be expressed as a sum of two matrices, with the π-mode and c-mode terms separated as

$$[Y_{\rm ch}] = \begin{bmatrix} \dfrac{R_c Y_{\pi 1}}{R_c - R_\pi} & \dfrac{-Y_{\pi 1}}{R_c - R_\pi} \\ \dfrac{Y_{\pi 2}}{(R_c - R_\pi)/R_c R_\pi} & \dfrac{-R_\pi Y_{\pi 2}}{R_c - R_\pi} \end{bmatrix}_\pi + \begin{bmatrix} \dfrac{-R_\pi Y_{c1}}{R_c - R_\pi} & \dfrac{Y_{c1}}{R_c - R_\pi} \\ \dfrac{-Y_{c2}}{(R_c - R_\pi)/R_c R_\pi} & \dfrac{R_c Y_{c2}}{R_c - R_\pi} \end{bmatrix}_c \quad (35)$$

and

$$[Z_{\rm ch}] = \begin{bmatrix} \dfrac{Z_{\pi 1} R_c}{R_c - R_\pi} & \dfrac{-Z_{\pi 1}}{(R_\pi - R_c)/R_c R_\pi} \\ \dfrac{-Z_{\pi 2}}{R_c - R_\pi} & \dfrac{-R_\pi Z_{\pi 2}}{R_c - R_\pi} \end{bmatrix}_\pi + \begin{bmatrix} \dfrac{-Z_{c1} R_\pi}{R_c - R_\pi} & \dfrac{Z_{c1}}{(R_\pi - R_c)/R_c R_\pi} \\ \dfrac{Z_{c2}}{R_c - R_\pi} & \dfrac{R_c Z_{c2}}{R_c - R_\pi} \end{bmatrix}_c. \quad (36)$$

These matrices are symmetric and singular. From these matrices, each associated with a mode (m), the corresponding configuration-oriented models are obtained by using two wire transmission lines. The characteristic admittance of the transmission line with its one end connected to ports i and k is given by

$$Y_{ik_m} = -Y_{\rm ch}(i,k)_m \quad (37)$$

$$[Y_{\rm ch}] = \begin{bmatrix} \dfrac{Y_{a1}(R_{v1} - R_{v2}) - Y_{b1}R_{v2} + Y_{c1}R_{v1}}{2(R_{v1} - R_{v2})} & \dfrac{Y_{b1} - Y_{c1}}{R_{v1} - R_{v2}} & \dfrac{Y_{a1}(R_{v2} - R_{v1}) - Y_{b1}R_{v2} + Y_{c1}R_{v1}}{2(R_{v1} - R_{v2})} \\ \dfrac{-R_{v1}R_{v2}Y_{b2} + R_{v1}R_{v2}Y_{c2}}{2(R_{v1} - R_{v2})} & \dfrac{R_{v1}Y_{b2} - R_{v2}Y_{c2}}{(R_{v1} - R_{v2})} & \dfrac{-R_{v1}R_{v2}Y_{b2} + R_{v1}R_{v2}Y_{c2}}{2(R_{v1} - R_{v2})} \\ \dfrac{-(R_{v1} - R_{v2})Y_{a3} - R_{v2}Y_{b3} + R_{v1}Y_{c3}}{2(R_{v1} - R_{v2})} & \dfrac{Y_{b3} - Y_{c3}}{R_{v1} - R_{v2}} & \dfrac{-(R_{v2} - R_{v1})Y_{a3} - R_{v2}Y_{b3} + Y_{c3}R_{v1}}{2(R_{v1} - R_{v2})} \end{bmatrix}. \quad (39)$$

and the characteristic admittance of the transmission line connecting port i and ground (0) is

$$Y_{i0_m} = \sum_{k=1}^{n} Y_{\mathrm{ch}}(i,k)_m. \tag{38}$$

The length of transmission lines corresponds to the electrical length of mode. Therefore, the matrices $[Y_{\mathrm{ch}}]$ and $[Z_{\mathrm{ch}}]$ can be synthesized with six transmission lines leading to the same equivalent circuits, as shown in Fig. 1(a) and (b).

The characteristic admittance matrix of three symmetrical coupled lines is derived in a similar manner in terms of the line-mode admittance of three lines, the mode voltages and current ratios and propagation constants for the normal modes, and is given by (39), shown at the bottom of the previous page [14], where Y_{a1}, Y_{a2}, $\cdots$, etc. are the line mode admittances of three lines for modes a, b, and c, and the corresponding voltage and current eigenvector matrices are defined in (17). For three symmetric coupled lines, $Y_{a1} = Y_{a3}$, $Y_{b1} = Y_{b3}$, and $Y_{c1} = Y_{c3}$. This matrix can be expressed as sum of three matrices for each mode and is readily realized leading to the transmission-line network of 13 transmission lines in a six-port network.

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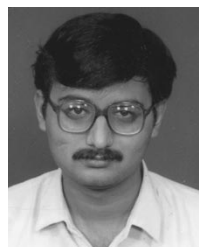

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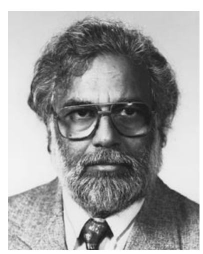

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